

Vector Fields
Conservative Fields

Question

The function \underline{F} is given by $\underline{F} = r \sin 2\theta \hat{r} + r \cos 2\theta \hat{\theta}$. Show that \underline{F} is conservative, and find a corresponding potential.

Answer

As $\underline{F} = r \sin 2\theta \hat{r} + r \cos 2\theta \hat{\theta} = \nabla \phi(r, \theta)$ we must have

$$\frac{\partial \phi}{\partial r} = r \sin(2\theta), \quad \frac{1}{r} \frac{\partial \phi}{\partial \theta} = r \cos(2\theta).$$

These are both satisfied if

$$\phi(r, \theta) = \frac{1}{2} r^2 \sin(2\theta) + C.$$

So \underline{F} is conservative, having ϕ as a potential.