Vector Fields
Conservative Fields

Question
For the following vector field, find whether it is conservative. If so, find a corresponding potential
$\underline{F}(x,y) = \dfrac{x\underline{i} + y\underline{j}}{x^2+y^2}$
Answer

$$\begin{aligned} F_1 &= \frac{x}{x^2+y^2} \\ F_2 &= \frac{y}{x^2+y^2} \\ \Rightarrow \frac{paF_1}{\partial y} &= -\frac{2xy}{(x^2+y^2)^2} = \frac{\partial F_2}{\partial x}. \end{aligned}$$

$\Rightarrow \underline{F}$ can be conservative.
If $\underline{F} = \nabla\phi$

$$\begin{aligned} \Rightarrow \frac{\partial\phi}{\partial x} &= \frac{x}{x^2+y^2} \\ \frac{\partial\phi}{\partial y} &= \frac{y}{x^2+y^2} \\ \Rightarrow \phi(x,y) &= \int \frac{x}{x^2+y^2}\,dx \\ &= \frac{\ln(x^2+y^2)}{2} + C_1(y) \\ \frac{y}{x^2+y^2} &= \frac{\partial\phi}{\partial y} = \frac{y}{x^2+y^2} + c_1'(y) \\ \Rightarrow c_1'(y) &= 0 \end{aligned}$$

So choose $C_1 y = 0$

$$\Rightarrow \phi(x,y) = \frac{1}{2}\ln(x^2+y^2)$$

is a scalar potential for $\underline{F}$, with $\underline{F}$ being conservative everywhere on $\Re^2$ except for the origin.