

Vector Fields
Conservative Fields

Question

For the following vector field, find whether it is conservative. If so, find a corresponding potential

$$\underline{F}(x, y, z) = x\underline{i} - 2y\underline{j} + 3z\underline{k}$$

Answer

$$F_1 = x, F_2 = -2y, F_3 = 3z.$$

$$\begin{aligned}\frac{\partial F_1}{\partial y} &= 0 = \frac{\partial F_2}{\partial x} \\ \frac{\partial F_1}{\partial z} &= 0 = \frac{\partial F_3}{\partial x} \\ \frac{\partial F_2}{\partial z} &= 0 = \frac{\partial F_3}{\partial y}\end{aligned}$$

$\Rightarrow \underline{F}$ may be conservative.

If $\underline{F} = \nabla\phi$

$$\Rightarrow \frac{\partial\phi}{\partial x} = x, \quad \frac{\partial\phi}{\partial y} = -2y, \quad \frac{\partial\phi}{\partial z} = 3z.$$

So $\phi(x, y, z) = \frac{x^2}{2} - y^2 + \frac{3z^2}{2}$ is a potential for \underline{F} , and so \underline{F} is conservative on \mathbb{R}^3 .