QUESTION

Solve the following linear programming problem using the bounded variable simplex method.

$$\begin{array}{ll} \text{Maximize} & z = -7x_1 + 2x_2 + 7x_3 - x_4 \\ \text{subject to} & 4x_1 - x_2 + x_3 + 2x_4 \le 8 \\ & 6x_1 + 3x_2 + 2x_3 - 5x_4 \le 25 \\ & 0 \le x_1 \le 1 \\ & 0 \le x_2 \le 11 \\ & 0 \le x_3 \le 9 \\ & 0 \le x_4 \le 5. \end{array}$$

(i) For the first constraint, give the range for the right-hand side within which the optimal basis remains unaltered. Also, perform this ranging analysis for the upper bound constraint $x_4 \le 5$.

(ii) If the objective function coefficient of x_2 changes to $2+\delta$, for what range of values of δ is the change in the maximum value of z proportional to δ?

ANSWER

Basic	z	x_1	x_2	x_3	x_4	s_1	s_2		Ratio
s_1	0	4	-1	1	2	1	0	8	8
s_2	0	6	3	2	-5	0	1	25	$\frac{25}{2}$
	1	7	-2	-7	1	0	0	0	

Basic	z	x_1	x_2	x_3	x_4	s_1	s_2		Ratio
x_3	0	4	-1	1	2	1	0	8	1
s_2	0	-2	5	0	-9	-2	1	9	$\frac{9}{5}$
	1	35	-9	0	15	7	0	56	

Perform simplex iteration and substitute $x_3' = 9 - x_3$.

Basic	z	x_1	x_2	x_3'	x_4	s_1	s_2		Ratio
x_2	0	-4	1	1	-2	-1	0	$-8+9=1$	
s_2	0	18	0	-5	1	3	1	$49-45=4$	
	1	-1	0	9	-3	-2	0	$-16+81=65$	

Basic	z	x_1	x_2	x_3'	x_4	s_1	s_2		
x_2	0	32	1	-9	0	5	1	9	$\frac{2}{9}$
x_4	0	18	0	-5	1	3	1	4	$\frac{1}{5}$
	1	53	0	-6	0	7	3	77	

Perform simplex iteration and substitute $x_4' = 5 - x_4$

Basic	z	x_1	x_2	x_3'	x_4'	s_1	s_2	
x_2	0	$-\frac{2}{5}$	1	0	$\frac{9}{5}$	$-\frac{2}{5}$	$\frac{1}{5}$	$\frac{9}{5}+9=\frac{54}{5}$
$x_3;$	0	$-\frac{18}{5}$	0	1	$\frac{1}{5}$	$-\frac{3}{5}$	$-\frac{1}{5}$	$-\frac{4}{5}+1=\frac{1}{5}$
	1	$\frac{157}{5}$	0	0	$\frac{6}{5}$	$\frac{17}{5}$	$\frac{9}{5}$	$72\frac{1}{5}+6=78\frac{1}{5}$

Thus we have an optimal solution

$$x_1 = 0 \; x_2 = 10\frac{4}{5} \; x_3' = \frac{1}{5} \; x_4' = 0 \; x_3 = 8\frac{4}{5} \; x_4 = 5 \; z = 78\frac{1}{5}$$

(i) If the right hand side of the first constraint is $8 + \delta$, then the right hand sides in the final tableau are $\frac{54}{5} = \frac{2}{5}\delta \quad \frac{1}{5} - \frac{3}{5}\delta$

For non-negativity, $\delta \leq 27 \; \delta \leq \frac{1}{3}$.

For basic variables to be in the range $\begin{array}{ll} \frac{54}{5} - \frac{2}{5}\delta \leq 11 & \delta \geq -\frac{1}{2} \\ \frac{1}{5} - \frac{3}{5}\delta \leq 9 & \delta \geq -\frac{44}{3} \end{array}$

Thus, the range is $-\frac{1}{2} \leq \delta \leq \frac{1}{3}$.

If $x_4 \leq 5$ is replaced by $x_4 \leq 5 + \delta$, then right hand sides become $\frac{54}{5} + \frac{9}{5}\delta \quad \frac{1}{5} + \frac{1}{5}\delta$

For non-negativity, $\delta \geq -\frac{54}{9} \quad \delta \geq -1$

For basic variables to be in range $\begin{array}{ll} \frac{54}{5} + \frac{9}{5}\delta \leq 11 & \delta \leq \frac{1}{9} \\ \frac{1}{5} + \frac{1}{5}\delta \leq 9 & \delta \leq 44 \end{array}$

Thus, the range is $-1 \leq \delta \leq \frac{1}{9}$.

(ii) For the new coefficient, the coefficient in the z-row are

$$z + \left(\frac{157}{5} - \frac{2}{5}\delta\right) x_1 + \left(\frac{6}{5} + \frac{9}{5}\delta\right) x_4' + \left(\frac{17}{5} - \frac{2}{5}\delta\right) s_1 + \left(\frac{9}{5} + \frac{1}{5}\delta\right) s_2 = 78\frac{1}{5} + \frac{54}{5}\delta$$

Thus, we require that

$$\begin{array}{rcl} \delta & \leq & \frac{157}{2} \\ \delta & \geq & -\frac{2}{3} \\ \delta & \leq & \frac{17}{2} \\ \delta & \geq & -9 \end{array}$$

so the range is $-\frac{2}{3} \leq \delta \leq \frac{17}{2}$