

QUESTION

Find the eigenvalues and eigenvectors of the following matrices.

$$\begin{bmatrix} 6 & 1 & 1 \\ 1 & 6 & 1 \\ 3 & 3 & 6 \end{bmatrix} \quad \begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -4 \\ 1 & 0 & -4 \end{bmatrix}$$

For each matrix A write down where possible a matrix M such that $M^{-1}AM$ is diagonal and check that M works.

ANSWER

First matrix

Eigenvalue	4	Eigenvector	$\begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$
Eigenvalue	5	Eigenvector	$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$
Eigenvalue	9	Eigenvector	$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

Second matrix

Eigenvalue	1	Eigenvector	$\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$
Eigenvalue	$3 + \sqrt{2}$	Eigenvector	$\begin{bmatrix} -1 \\ \sqrt{2} \\ 1 \end{bmatrix}$
Eigenvalue	$3 - \sqrt{2}$	Eigenvector	$\begin{bmatrix} -1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$

Third matrix

Eigenvalue	0	Eigenvector	$\begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix}$
Eigenvalue	-3,-3	Eigenvector	$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ only

Where there are three independent eigenvectors the matrix M which has the eigenvectors as its columns will do.