QUESTION

If A and B are $n \times n$ matrices show that tr(AB)=tr(BA). Deduce that if two matrices are similar then their traces are equal.

ANSWER

$\mathrm{tr}(AB) = \sum_r (AB)_{rr} = \sum_r \sum_s (A)_{rs}(B)_{sr} = \sum_s \sum_r (B)_{sr}(A)_{rs} = \sum_s (BA)_{ss} = \mathrm{tr}(BA)$

Hence $\mathrm{tr}(A^{-1}BA) = \mathrm{tr}(A^{-1}(BA)) = \mathrm{tr}((BA)A^{-1}) = \mathrm{tr}(BAA^{-1}) = \mathrm{tr}(B)$.