Question

Twelve patients suffering from asthma are given treatment each evening to relieve their symptoms. They will assess the state of their breathlessness as nil, mild, moderate or severe (coded 0, 1, 2 and 3 respectively), on the morning before treatment starts (morning 0) and on each of the four mornings during the treatment period (1, 2, 3 and 4). The following data are obtained:

Morning	1	2	3	4	5	6	7	8	9	10	11	12
0	2	2	3	2	1	2	2	3	3	1	2	2
1	2	2	3	1	2	2	1	3	2	1	2	3
2	1	2	2	0	2	1	1	2	2	1	1	2
3	1	2	2	0	1	1	0	1	1	1	0	2
4	1	1	2	0	0	1	1	1	0	0	0	1

(i) Using the above table, count the number of transitions between each pair of states. From this information estimate the transition probability ,matrix, assuming breathlessness can be modelled by a homogeneous Markov chain. Explain the meaning of the assumption.

(ii) If a patient with moderate breathlessness is given the treatment, what does the Markov chain model give for the probability that his degree of breathlessness will be reduced on morning 1?

(iii) Obtain the distribution predicted by the model for the degree of breathlessness on day 1. Compare this with the frequency distribution observed on day 1. Explain, without performing any calculations, how the same comparison could be made for morning 4. Suggest another method of investigating the validity of the Markov assumption.

Answer

Counting transitions gives the following results:

	0	1	2	3	
0	3	1	0	0	4
1	6	9	1	0	16
2	0	10	11	1	22
3	0	0	4	2	6
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(i) Estimated P:

$$\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 & 0 \\ \frac{6}{16} & \frac{9}{16} & \frac{1}{16} & 0 \\ 0 & \frac{10}{22} & \frac{11}{22} & \frac{1}{22} \\ 0 & 0 & \frac{4}{6} & \frac{2}{6} \end{pmatrix}$$

The Markov property says that the probability of transition depends only on the current state, and not on previous history. Also it is time homogeneous i.e., it doesn't depend on which day is considered.

(ii) $P(< 2 \text{ on morning } 1 \mid 2 \text{ on morning } 0) = p_{20} + p_{21} = \dfrac{10}{22}$

(iii) The initial distribution is

$\underline{p}_0 = \dfrac{1}{12}\,(0, 2, 7, 3)$ from the table.

On morning 1 the predicted distribution is

$$\begin{aligned} \underline{p}_0 P &= \left(\frac{1}{16}, \frac{379}{1056}, \frac{15}{32}, \frac{29}{264}\right) \\ &= (.0625,\ .3589,\ .4688,\ .1098) \end{aligned}$$

the observed relative frequency on day 1 is

$$\frac{1}{12}(0, 3, 6, 3) = (0,\ .25,\ .5,\ .25)$$

On morning 4 the predicted distribution is $\underline{p}_0 p^4$, which could be compared with the observed frequency.

Alternatively we could look at

$$rel\ freq\ (X_n = k \mid X_{n-2} = i + X_{n-1} = j)$$

to test the Markov assumption.

e.g.

$rel\ freq\ (X_2 = 2 \mid X_1 = 2 \text{ and } \mathrm{X}_0 = 3) = 1$ different

$rel\ freq\ (X_2 = 2 \mid X_1 = 2 \text{ and } \mathrm{X}_0 = 2) = \dfrac{1}{4}$