

Multiple Integration
Iteration of Double Integrals

Question

Calculate the given double integrals by iteration

$$\iint_T \sqrt{a^2 - y^2} dA$$

With T being the triangle with vertices $(0, 0)$, $(a, 0)$ and (a, a) .

Answer

$$\begin{aligned} I &= \iint_T \sqrt{a^2 - y^2} dA \\ &= \int_0^a \sqrt{a^2 - y^2} dy \int_y^a dx \\ &= \int_0^a (a - y) \sqrt{a^2 - y^2} dy \\ &= a \int_0^a \sqrt{a^2 - y^2} dy - \int_0^a y \sqrt{a^2 - y^2} dy \\ &\quad \text{Let } u = a^2 - y^2 \\ &\quad \text{and } du = -2y dy \end{aligned}$$

$$\begin{aligned} \Rightarrow I &= a \frac{\pi a^2}{4} + \frac{1}{2} \int_{a^2}^0 u^{1/2} du \\ &= \frac{\pi a^3}{4} - \frac{1}{3} u^{3/2} \Big|_0^{a^2} \\ &= \left(\frac{\pi}{4} - \frac{1}{3} \right) a^4 \end{aligned}$$