Multiple Integration
Iteration of Double Integrals

Question
Find the volume of the given solid
Below $z = 1 - x^2 - y^2$ and over the region $x \geq 0$, $y \geq 0$, $x + y \leq 1$.
Answer

$$\begin{aligned} V &= \int_0^1 dx \int_0^{1-x} (1 - x^2 - y^2)\, dy \\ &= \int_0^1 \left((1 - x^2)y - \frac{y^3}{3} \right) \Bigg|_{y=0}^{y=1-x} dx \\ &= \int_0^1 \left((1 - x^2)(1 - x) - \frac{(1 - x)^3}{3} \right) dx \\ &= \int_0^1 \left(\frac{2}{3} - 2x^2 + \frac{4x^3}{3} \right) dx \\ &= \frac{2}{3} - \frac{2}{3} + \frac{1}{3} = \frac{1}{3} \text{cu. units} \end{aligned}$$