

Multiple Integration
Iteration of Double Integrals

Question

Calculate the given double integral by iteration.

$$\iint_R (x^2 + y^2) dA$$

With R being the area $0 \leq x \leq a$, $0 \leq y \leq b$.

Answer

$$\begin{aligned} \iint_R (x^2 + y^2) dA &= \int_0^a dx \int_0^b (x^2 + y^2) dy \\ &= \int_0^a dx \left(x^2 y + \frac{y^3}{3} \right) \Big|_{y=0}^{y=b} \\ &= \int_0^a \left(bx^2 + \frac{1}{3}b^3 \right) dx \\ &= \frac{1}{3} \left(bx^3 + b^3 x \right) \Big|_0^a \\ &= \frac{1}{3} (a^3 b + ab^3) \end{aligned}$$