

Multiple Integration
Iteration of Double Integrals

Question

Find the volume for the solid defined by

The space inside two cylinders, $x^2 + y^2 = a^2$ and $y^2 + z^2 = a^2$.

Answer

$$\begin{aligned} V &= 8 \times (\text{vol in first octant}) \\ &= 8 \int_0^a dx \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2} dy \\ &= 8 \int_0^a (a^2-x^2) dx \\ &= 8 \left(a^2x - \frac{x^3}{3} \right) \Big|_0^a \\ &= \frac{16}{3} a^3 \text{ cu. units} \end{aligned}$$