

Multiple Integration
Iteration of Double Integrals

Question

Calculate the given double integral by iteration in the region defined by the given curves.

$$\iint_D x \cos y \, dA$$

With D being the region bounded by $y = 1 - x^2$ and the coordinate axes, in the first quadrant.

Answer

$$\begin{aligned} I &= \iint_D x \cos y \, dA \\ &= \int_0^1 x \, dx \int_0^{1-x^2} \cos y \, dy \\ &= \int_0^1 x \, dx (\sin y) \Big|_{y=0}^{y=1-x^2} \\ &= \int_0^1 x \sin(1 - x^2) \, dx \end{aligned}$$

$$\text{Let } u = 1 - x^2$$

$$\Rightarrow du = -2x \, dx$$

$$\begin{aligned} \Rightarrow I &= -\frac{1}{2} \int_0^1 \sin u \, du \\ &= \frac{1}{2} \cos u \Big|_1^0 \\ &= \frac{1 - \cos(1)}{2} \end{aligned}$$