

Multiple Integration
Iteration of Double Integrals

Question

Calculate the given double integral by iteration in the region defined by the given curves.

$$\iint_R xy^2 dA$$

With R being the region bounded by $y = x^2$ and $x = y^2$ in the first quadrant.

Answer

$$\begin{aligned}\iint_R xy^2 dA &= \int_0^1 x dx \int_{x^2}^{\sqrt{x}} y^2 dy \\ &= \int_0^1 x dx \left(\frac{1}{3} y^3 \right) \Big|_{y=x^2}^{y=\sqrt{x}} \\ &= \frac{1}{3} \int_0^1 (x^{5/2} - x^7) dx \\ &= \frac{1}{3} \left(\frac{2}{7} x^{7/2} - \frac{x^8}{8} \right) \Big|_0^1 \\ &= \frac{1}{3} \left(\frac{2}{7} - \frac{1}{8} \right) = \frac{3}{56}\end{aligned}$$