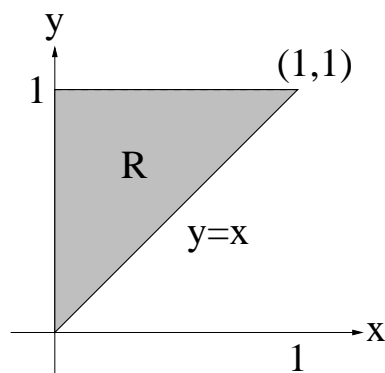# *Iteration of Double Integrals*

Question

Sketch the domain of integration, and calculate the iterated integral for

$$\int_0^1 dx \int_x^1 \frac{y^\lambda}{x^2 + y^2} \, dy \quad (\lambda > 0)$$

Answer

$$
\begin{aligned}
I &= \int_0^1 dx \int_x^1 \frac{y^\lambda}{x^2 + y^2} \, dy \quad (\lambda > 0) \\
&= \iint_R \frac{y^\lambda}{x^2 + y^2} \, dA \\
&= \int_0^1 y^\lambda \, dy \int_0^y \frac{dx}{x^2 + y^2} \\
&= \int_0^1 y^\lambda \, dy \frac{1}{y} \left(\tan^{-1} \frac{x}{y} \right) \Big|_{x=0}^{x=y} \\
&= \frac{\pi}{4} \int_0^1 y^{\lambda - 1} \, dy \\
&= \frac{\pi y^\lambda}{4\lambda} \Big|_0^1 = \frac{\pi}{4\lambda}
\end{aligned}
$$

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