Question

A simple pendulum has angular position θ and angular momentum p. The motion of the pendulum (assuming a suitable set of measurement units so there are no constants in the equation) can then be described by the following ordinary differential equation for $p(\theta)$

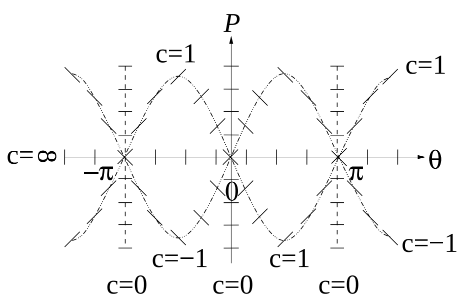
$$\frac{dp}{d\theta} = -\frac{\sin\theta}{p} \; .$$

Sketch the direction field (note the periodicity and show values of $-2\pi \leq \theta \leq 2\pi$). Comment on the different behaviour between a solution that has a very small value of p when $\theta = 0$ and a solution that has very large p when $\theta = 0$. $\quad (*)$

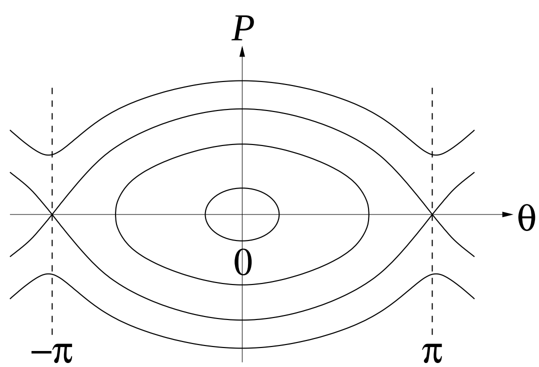
Answer

Isoclines are $c = -\frac{\sin\theta}{P}$.

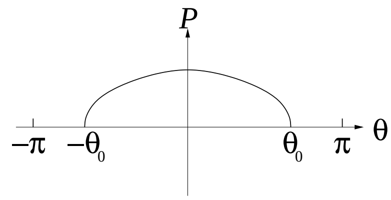
$$\Rightarrow -cP = \sin\theta$$



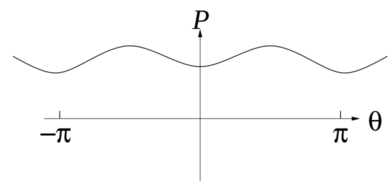
$c = 0$	$\theta = n\pi$
	$n = 0, 1, 2 \cdots$
$c = 1$	$P = -\sin\theta$
$c = 2$	$P = \sin\theta$
$c \to \infty$	$P = 0$



If P is small when $\theta = 0$ then the solution $P(\theta)$ only exists for values of θ, $-\pi < -\theta_0 \leq \theta \leq \theta_0 < \pi$. e.g.



(A pendulum oscillating back and forth)
If P is large when $\theta = 0$ then the solution $P(\theta)$ exists for all θ. e.g.



(A pendulum swinging over and over).