

Question

For each of the following one-parameter families of functions, find a first-order differential equation (**not** containing the constant c) which is satisfied by all members of the family. In each case sketch graphs of a few typical functions in the family.

1. $y = ce^{-2x}$ (*)

2. $y = cx^2 - x$

3. $y = (c + x)e^{3x}$ (*)

Answer

1.

$$\begin{aligned}y = ce^{-2x} &\Rightarrow \frac{dy}{dx} = -2ce^{-2x} \\ \text{eliminate } c & \\ &\Rightarrow \\ ye^{2x} = c &\Rightarrow \frac{dy}{dx} = -2(ye^{2x})e^{-2x} \\ &\Rightarrow \frac{dy}{dx} = -2y\end{aligned}$$

2.

$$\begin{aligned}y = cx^2 - x &\Rightarrow \frac{dy}{dx} = 2cx - 1 \\ &\Rightarrow c = \frac{y + x}{x^2} \\ &\Rightarrow \frac{dy}{dx} = 2\left(\frac{y + x}{x^2}\right)x - 1 \\ &= 2\left(\frac{y + x}{x}\right) - 1 \\ &= 2\left(\frac{y}{x} + 1\right) - 1 \\ &= 2\frac{y}{x} + 1\end{aligned}$$

3.

$$y = (c + x)e^{3x}$$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= 3(c + x)e^{3x} + e^{3x} \\ &= 3ce^{3x} + (3x + 1)e^{3x}\end{aligned}$$

$$\Rightarrow c = e^{-3x}y - x$$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= 3(e^{-3x}y - x) + (3x + 1)e^{3x} \\ &= 3y + e^{3x}\end{aligned}$$