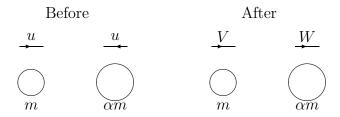**Question**

Two particles, one of mass m and one of mass αm ($\alpha > 0$), both have speed U. They suffer a head-on collision. What are their speeds after the collision of their coefficient of restitution is e?

Answer

Before   After

u u V W

m αm m αm

Conservation of mometum

$$mU - \alpha mU = mV + \alpha mW$$
$$U(1 - \alpha) = V + \alpha W \quad (1)$$

Newton's law of restitution

$$-e2U = V - W \quad (2)$$
$$(1) - (2)$$
$$u(1 - \alpha + 2e) = \alpha W + W$$
$$W = \frac{u(1 - \alpha + 2e)}{1 + \alpha}$$

sub into (1) and rearrange

$$V = u(1 - \alpha) - \frac{\alpha u(1 - \alpha + 2e)}{1 + \alpha}$$
$$= \frac{u(1 - \alpha)(1 + \alpha) - u\alpha(1 - \alpha + 2e)}{1 + \alpha}$$
$$= \frac{u(1 - \alpha(1 + 2e))}{1 + \alpha}$$

Thus the answers are:

$$W = \frac{u(1 - \alpha + 2e)}{1 + \alpha}; \quad V = \frac{u(1 - \alpha(1 + 2e))}{1 + \alpha}$$