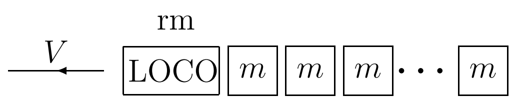
Question

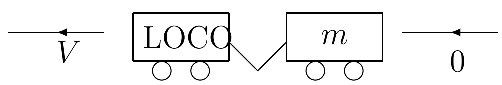
A set of n wagons, each of mass m, standing in a railway siding is set in motion by a locomotive of mass rm starting with speed V. Initially the couplings between the wagons are slack, as is the coupling between the first wagon and the locomotive, and each wagon moves a distance s before it jerks the succeeding wagon in motion. The coefficient of restitution of the coupling is e. Find the time which ensues between the movement of the first and last wagons, neglecting the effect of friction and assuming that each coupling does not remain taut until the last wagon moves.

Answer

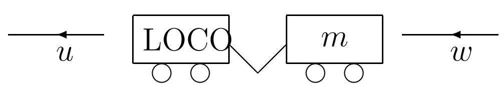


Consider the locomotive and the first wagon.

Before the coupling is taut:



After the coupling is jerked:



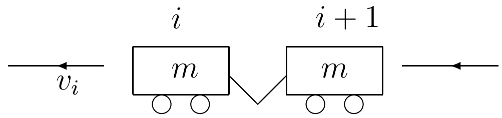
Using the law of conservation of momentum

$$\begin{aligned} rmV + 0 &= rmu + mw \\ rV &= ru + w \qquad (1) \\ \text{Restitution law : } \quad -eV &= u - w \qquad (2) \\ \text{Solving (1) and (2) gives : } \quad w &= rV\frac{1+e}{1+r} \qquad (3) \end{aligned}$$

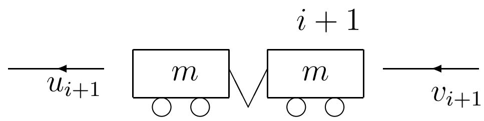
Note that equation (2) implies that $w > u$, so the coupling is slack after the wagon is jerked forward.

Consider the i and $i+1^{th}$ wagon.

Before:



After:



The same procedure gives (3) with $r=1, v=v_i$
Therefore $v_{i+1}=v_i\frac{1+e}{2}$ (Wagons successively move more slowly as $\frac{1+e}{2}<1$)
If T$_i$ is the time taken to elapse between wagon i and wagon $i+1$ starting to move then the total time for the last wagon to move is
$T=T_1+T_2+...+T_{n-1}$
$T=T_1+\alpha T_1+\alpha^2 T_1+...+\alpha^{n-2}T_1$
Where $\alpha=\frac{2}{1+e}$ because the time is the distance moved by wagon(s) divided by the speed v_i.
Thus $T_1=\frac{s}{w}=\frac{s(1+r)}{V(1+e)r}$

$$T=\frac{s}{Vr}\left(\frac{1+r}{1+e}\right)\left(\frac{1-\alpha^{n-1}}{1-\alpha}\right)=\frac{s}{Vr}\cdot\frac{1+r}{1+e}\cdot\frac{(1+e)^{n-1}-2^{n-1}}{(e-1)(1+e)^{n-1}}$$