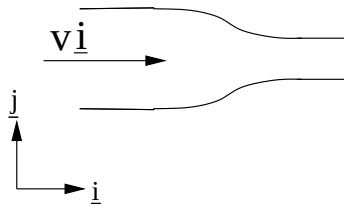
Question

A pipe has a variable circular cross section; the radius r depends on the distance along the pipe, x, as $r = a - be^{-x}$, with $a > b$. If the speed of the water in the pipe is v_0 at the inlet ($x = 0$) find the speed of the water in the pipe as a function of x.

Answer

Mass Flux: $\mathbf{j} = \rho v(x)\mathbf{i}$



Mass is conserved, therefore

$$\frac{dm}{dt} = -\int_{S+} \mathbf{j}\cdot\mathbf{n}\,ds + \int_{S-} \mathbf{j}\cdot\mathbf{n}\,ds$$

Now $\dfrac{dm}{dt} = 0$. Thus $\displaystyle\int_{x=0} \mathbf{j}\cdot\mathbf{n}\,ds = \int_{\alpha} \mathbf{j}\cdot\mathbf{n}\,ds$ as ρ is constant.

Therefore $\rho\, 2\pi(a-b)v_0 = \rho 2\pi(a - be^{-x})v(x) \Rightarrow v(x) = \dfrac{a-b}{a-be^{-x}}$