QUESTION

Prove that, for any prime p, if $p|a^2n$ then $p|a$ and hence $p^n|a^n$.

ANSWER

Suppose a is written as a product of prime powers as $p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_k^{\alpha_k}$. Then $a^n = p_1^{n\alpha_1}p_2^{n\alpha_2}\ldots p_k^{n\alpha_k}$. Now $p|a^n$, so p must be equal to one of the primes $p_1, p_2 \ldots p_k$. Thus $p|a$ and $p^n|a^n$, as required.