Question
Answer
The covariance matrix σ_i, must be positive definite as assets cannot have arbitrary covariances. For example, it is evidently impossible to have 3 risky assets where all the covariances are -1 as 3 assets cannot all move independently in opposite directions to each other.
Now consider the portfolio $\Pi = \lambda_1 S_1 + \lambda_2 S_2 + \lambda_3 S_3$ where the λ_i sum to 1 (but need not be between 0 and 1).
We have

$$\overline{R}_\Pi = \frac{1}{100}(10\lambda_1 + 12\lambda_2 + 15\lambda_3)$$

$$\begin{aligned}
\sigma_\Pi^2 &= \sum_{i=1}^{3}\sum_{j=1}^{3}\sigma_{ij}\lambda_i\lambda_j \\
&= \frac{1}{100^2}(9\lambda_1^2 + 25\lambda_2^2 + 100\lambda_3^2 + 12\lambda_1\lambda_2 - 12\lambda_1\lambda_3 + 20\lambda_2\lambda_3)
\end{aligned}$$

An investment consisting of Π and cash along the straight line joining $(0.06, 0)$ to $(\overline{R}_\Pi, \sigma_\Pi)$ in the Risk/Reward diagram. This line has slope

$$\theta = \frac{\overline{R}_\Pi - 0.06}{\sigma_\Pi}$$

$$\theta = \frac{\overline{R}_\Pi - 0.06}{\sigma_\Pi} = \frac{(10\lambda_1 + 12\lambda_2 + 15\lambda_3) - 6(\lambda_1 + \lambda_2 + \lambda_3)}{(9\lambda_1^2 + 25\lambda_2^2 + 100\lambda_3^2 + 12\lambda_1\lambda_2 - 12\lambda_1\lambda_3 + 20\lambda_2\lambda_3)^{\frac{1}{2}}}$$

Let $A =$ the bottom $\Rightarrow\ \theta = \frac{4\lambda_1 + 6\lambda_2 + 9\lambda_3}{A}$

$$\begin{aligned}
\frac{\partial\theta}{\partial\lambda_1} &= \frac{4}{A} - \frac{1}{2}\left(\frac{4\lambda_1 + 6\lambda_2 + 9\lambda_3}{A^3}\right)(18\lambda_1 + 12\lambda_2 - 12\lambda_3) = 0 \\
\frac{\partial\theta}{\partial\lambda_2} &= \frac{6}{A} - \frac{1}{2}\left(\frac{4\lambda_1 + 6\lambda_2 + 9\lambda_3}{A^3}\right)(50\lambda_2 + 12\lambda_2 + 20\lambda_3) = 0 \\
\frac{\partial\theta}{\partial\lambda_3} &= \frac{9}{A} - \frac{1}{2}\left(\frac{4\lambda_1 + 6\lambda_2 + 9\lambda_3}{A^3}\right)(20\lambda_2 - 12\lambda_2 + 20\lambda_3) = 0
\end{aligned}$$

Now multiply through by $2A$, put $\alpha = (4\lambda_1 + 6\lambda_2 + 9\lambda_3)/A^2$ and we get (justified by the usual argument)

$$\begin{aligned}
18\lambda_1 + 12\lambda_2 - 12\lambda_3 &= 8/\alpha \\
50\lambda_2 + 12\lambda_1 + 20\lambda_3 &= 12/\alpha \\
200\lambda_3 - 12\lambda_1 + 20\lambda_2 &= 18/\alpha
\end{aligned}$$

now put $u = \alpha\lambda_1$, $v = \alpha\lambda_2$, $w = \alpha\lambda_3$

$$\begin{aligned} 18u + 12v - 12w &= 8 & \Rightarrow \quad 12u + 8v - 8w &= 16/3 \\ 12u + 50v + 20w &= 12 & \Rightarrow \quad 70v + 220w &= 30 \\ -12u + 20v + 200w &= 18 & \Rightarrow \quad 28v + 192w &= 70/3 \end{aligned}$$

$\Rightarrow$ finally

$$u = \frac{251}{546}, \quad v = \frac{47}{546}, \quad w = \frac{17}{156}$$

$$u + v + w = \frac{55}{84} = \alpha(\lambda_1 + \lambda_2 + \lambda_3) = \alpha$$

$\Rightarrow \ \lambda_1 = \dfrac{502}{715}, \quad \lambda_2 = \dfrac{94}{715}, \quad \lambda_3 = \dfrac{119}{715}$ ($\Rightarrow$ no short selling.)

$$\begin{aligned} \Rightarrow \overline{R}_\Pi &= 11.095\% \\ \sigma_\Pi &= 2.790 \\ \theta &= \frac{\overline{R}_\Pi - 6/100}{\sigma_\Pi} \sim 1.8265 \end{aligned}$$