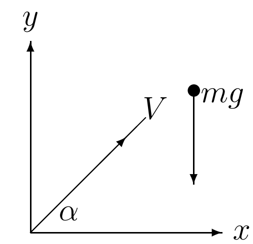
Question

A projectile moves under gravity near the earths surface such that its distance from its point of projection is always increasing. Find the maximum angle above the horizontal with which the particle could have been thrown. (Assume no air resistance).

Answer



Use Newton's second law and integrate to obtain
$x = (V\cos\alpha)t$ and $y = (V\sin\alpha)t - \frac{1}{2}gt^2$
The Distance of the particle from 0 is $r = \sqrt{x^2+y^2}$
We require that r is always increasing with time: $\dot{r} = \dfrac{x\dot{x}+y\dot{y}}{r} > 0$
Now $x\dot{x}+y\dot{y} = \dfrac{1}{2}g^2t^3 - \dfrac{3}{2}gV\sin\alpha t^2 + V^2t$
Now $\dot{r} = 0$ when $t = \dfrac{3}{2}\dfrac{V\sin\alpha}{g} \pm \dfrac{V}{2g}\sqrt{9\sin^2\alpha - 8}$
We want to choose α so that no such t exists i.e. to ensure that the above t is not real. This occurs when $9\sin^2\alpha < 8$ Thus $\alpha < 70.8°$