QUESTION

A pair of unbiased coins is tossed repeatedly until 2 heads are obtained. What is the probability that it will happen on the r-th toss? What is the probability that it will take more then two tosses? Find the mean and variance of the number of tosses.

ANSWER

p(2 heads with pair of unbiased coins)$=\frac{1}{4}$.

Hence the number of trials, r, has a geometric distribution

$p(r) = (\frac{3}{4})^{r-1}\frac{1}{4}$

p(more than 2 tosses) $= p$(first and second tosses fail) $= (\frac{3}{4})^2 = \frac{9}{16}$

$$\begin{aligned}\sum_{r=1}^{\infty} p^r &= \frac{p}{1-p} \text{ differentiating}\\ \sum_{r=1}^{\infty} rp^{r-1} &= \frac{1}{(1-p)^2}\\ \sum_{r=2}^{\infty} r(r-1)p^{r-2} &= \frac{2}{(1-p)^3}\end{aligned}$$

$$\begin{aligned}\mu &= \sum_{r=1}^{\infty} \frac{r}{4}\left(\frac{3}{4}\right)^{r-1}\\ &= \frac{\frac{1}{4}}{\left(1-\frac{3}{4}\right)^2} = 4\end{aligned}$$

$$\begin{aligned}\sigma^2 &= \sum_{r=1}^{\infty} \frac{r(r-1)}{4}\left(\frac{3}{4}\right)^{r-1} + \sum_{r=1}^{\infty} \frac{r}{4}\left(\frac{3}{4}\right)^{r-1} - \mu^2\\ &= \frac{1}{4} \times \frac{2\times\frac{3}{4}}{\left(1-\frac{3}{4}\right)^3} + 4 - 16\\ &= 24 + 4 - 16 = 12\end{aligned}$$