QUESTION

An electrical shop replenishes its supply of a specialized type of bulb at the beginning of each week. The demand in a week for this type of bulb has a Poisson distribution with mean 3.

Find the smallest number of bulbs which should be in stock at the beginning of the week to be at least 90% confident of satisfying the demand in a week. If the number in stock at the beginning of the week is set at that number find the expected number of satisfied demands in the week.

ANSWER

Demand $\sim P(3)$. We can use either $p(x) = \dfrac{\mathrm{e}^{-3}3^x}{x!}$ or tables to find $F(x)$.

$$\begin{array}{ll} p(0) = .050 & F(0) = .050 \\ p(1) = .149 & F(1) = .199 \\ p(2) = .224 & F(2) = .423 \\ p(3) = .224 & F(3) = .647 \\ P(4) = .168 & F(4) = .815 \\ & F(5) = .916 \end{array}$$

(Note that there are two modes, 2 and 3.)

Hence the least number to be at least 90% certain is 5. So stock 5. If the demand is $0, 1, 2, 3, 4$ then the satisfied demand is $0, 1, 2, 3, 4$. If the demand is ≥ 5 then the satisfied demand is 5.

E(satisfied demand)

$= 0p(0) + 1p(1) + 2p(2) + 3p(3) + 4p(4) + 5(p(5) + p(6) + \ldots)$

$= 0p(0) + 1p(1) + 2p(2) + 3p(3) + 4p(4) + 5(1 - p(4))$

$= .149 + .448 + .672 + .672 + 5 \times .185 = 2.866$