

QUESTION

A contractor rents out a piece of heavy equipment for t hours and is paid £50 per hour. The equipment tends to overheat and if it overheats x times during the hiring period the contractor will have to pay a repair cost x^2 . The number of times the equipment overheats in t hours can be assumed to have a poisson distribution with mean $2t$. What value of t will maximize the expected profit of the contractor?

ANSWER

If the equipment is hired for time t and has x faults then

$$P = 50t - x^2$$

$$E(P) = 50t - E(X^2)$$

$$X \sim P(2t) \text{ therefore}$$

$$\begin{aligned} E(X) &= 2t \\ \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 2t - 4t^2 \\ E(X^2) &= 2t + 4t^2 \end{aligned}$$

$$\begin{aligned} E(P) &= 50t - 2t - 4t^2 \\ &= 48t - 4t^2 \\ \frac{dE(P)}{dt} &= 48 - 8t = 0 \text{ when } t = 6 \\ \frac{d^2E(P)}{dt^2} &= -8 < 0 \text{ hence maximum.} \end{aligned}$$