

**Question**

Use Lagrange's method to find the general solution of  $yuu_x + xuu_y = xy$ .

**Answer**

Lagrange gives

$$\begin{aligned} \frac{dx}{d\xi} &= yu, \quad \frac{dy}{d\xi} = xu, \quad \frac{du}{d\xi} = xy \\ \Rightarrow \frac{dy}{dx} &= \frac{x}{y} \quad \text{and} \quad \frac{du}{dy} = \frac{y}{u} \\ \Rightarrow y^2 - x^2 &= \text{const} \quad \text{and} \quad u^2 - y^2 = \text{const} \\ \Rightarrow \text{the general solution is:} \end{aligned}$$

$$u^2 - y^2 = f(x^2 - y^2)$$

$$\Rightarrow \underline{u^2 = y^2 + f(x^2 - y^2)}$$