Question

The random pair X and Y has the distribution

		y			
		2	3	4	Total
	1	$\frac{1}{12}$	$\frac{1}{6}$	0	–
x	2	$\frac{1}{6}$	0	$\frac{1}{3}$	–
	3	$\frac{1}{12}$	$\frac{1}{6}$	0	–
		–	–	–	1

(a) Are X and Y independent? Give reasons.

(b) Find the conditional pmf of Y given that $X = 2$.

Hence find $E(Y|X = 2)$.

Answer

The probability table is

		y			
		2	3	4	Total
	1	$\frac{1}{12}$	$\frac{1}{6}$	0	$\frac{1}{4}$
x	2	$\frac{1}{6}$	0	$\frac{1}{3}$	$\frac{1}{2}$
	3	$\frac{1}{12}$	$\frac{1}{6}$	0	$\frac{1}{4}$
		$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1

(a) Since $P(X = 1, Y = 4) \neq P(X = 1) \cdot P(Y = 4)$ $(0 \neq \frac{1}{4} \cdot \frac{1}{3})$

X and Y are dependent.

(b) The conditional distribution of $Y|X = 2$ is

y	$f(y\|X = 2)$	
2	$\frac{1}{6} \div \frac{1}{2}$	$\frac{1}{3}$
3	$0 \div \frac{1}{3}$	0
4	$\frac{1}{3} \div \frac{1}{2}$	$\frac{2}{3}$

$$E(Y|X = 2) = 2 \cdot \frac{1}{3} + 3 \cdot 0 + 4 \cdot \frac{2}{3} = \frac{10}{3}.$$