**Question**
Consider the element of arc-length $\frac{1}{2+5|z|}|\mathrm{dz}|$ on $\mathbf{C}$. Calculate the circumference, with respect to this element of arc-length, of the Euclidean circle with center 0 and radius $r > 0$.

Further, write down the integral for the length, with respect to this element of arc-length, of the square with vertices at $\pm r \pm ri$ for $r > 0$. Evaluate it if you can.

**Answer**
$C_r$ the circle with center 0 and radius $r$. Parametrize $C_r$ by $f(t) = re^{it} \quad 0 \leq t \leq t\pi$.
Then,

$$\begin{aligned} f'(t) &= \int_f \frac{|\,dz|}{2+5|z|} \\ &= \int_0^{2\pi} \frac{|f'(t)|\,dt}{2+5|f(t)|} \\ &= \int_0^{2\pi} \frac{r\,dt}{2+5r} = \frac{2\pi r}{2+5r} \end{aligned}$$

Parametrize the side of the square $S_r$ from $r - ri$ to $r + ri$ by $f(t) = r + ti,\ -r \leq t \leq r$.
Then, $f'(t) = i$ and so

$$\begin{aligned} \mathrm{length(S_r)} &= 4\ \mathrm{length(side)} \\ &= 4\int_f \frac{|\,dz|}{2+5|z|} \\ &= 4\int_{-r}^{r} \frac{|f'(t)|\,dt}{2+5|f(t)|} \\ &= 4\int_{-r}^{r} \frac{dt}{2+5\sqrt{r^2+t^2}} \end{aligned}$$

(The fact that $\dfrac{1}{2+5|z|}|\,dz|$ is invariant by notations of $\mathbf{C}$ fixing 0 is what allows us to say that the length of the square is 4 times the length of one side.)
(I don't know how to evaluate this integral and I haven't yet found it in a table.)