

### Exam Question

#### Topic: TripleIntegral

A solid circular cylinder has radius 2, and the distance between its circular ends is 6. The density at a point  $P$  of the cylinder is proportional to the product of the square of the distance of  $P$  from the axis of the cylinder and the distance of  $P$  from the nearest circular end of the cylinder. Find the total mass, and the average density of the cylinder.

**Solution** In cylindrical polars

$$\begin{aligned} M &= 2k \int_0^{2\pi} d\phi \int_0^3 dz \int_0^2 r^2 z \cdot r dr = 4\pi k \int_0^3 z dz \int_0^2 r^3 dr \\ &= 4\pi k \frac{9}{2} \frac{16}{4} = 72\pi k. \end{aligned}$$

the volume of the cylinder is  $\pi \cdot 2^2 \cdot 6 = 24\pi$ .

So the average density is  $\frac{72\pi k}{24\pi} = 3k$ .