

QUESTION

Find $u(W, t)$ and $V(W, t)$, where

$$dX = udt + v dW$$

and

(i) $X(t) = W(t)^2$

(ii) $X(t) = 1 + t + \exp(W(t))$

(iii) $X(t) = f(t)W(t)$, f bounded and continuous.

ANSWER

Itô:

$$dx = \frac{\partial x}{\partial t} dt + \frac{\partial x}{\partial w} dw + \frac{1}{2} \frac{\partial^2 x}{\partial w^2} dt$$

(i)

$$\begin{aligned} dx &= 0dt + 2wdw + \frac{1}{2}2dt \\ dx &= dt + 2wdw \end{aligned}$$

(ii)

$$\begin{aligned} dx &= 1dt + e^w dw + \frac{1}{2}e^w dt \\ dw &= \left(1 + \frac{e^w}{2}\right) dt + e^w dw \end{aligned}$$

(iii)

$$\begin{aligned} dx &= w \frac{df}{dt} dt + f dw + \frac{1}{2}0dt \\ dx &= w f' dt + f dw \end{aligned}$$