QUESTION

(i) Evaluate the double integral $\int_0^2 \int_{y^2}^{2y} (4x - y)\, dx dy$.

(ii) Given the integral $\int_0^4 \int_{\sqrt{y}}^{2} y \cos x^5 \, dx dy$, sketch the region of integration.

Reverse the order of integration and evaluate the resulting integral.

(iii) Sketch the plane region R defined by the inequalities:

$$y \leq \sqrt{x}, \; y \geq \sqrt{3x - 18}, \; x \geq 0.$$

By choosing an appropriate order of integration show how to express an integral of the form $\iint_R f(x, y)\, dA$ as a double integral, computing the limits.

ANSWER

(i)

$$\begin{aligned}
\int_0^2 \int_{y^2}^{2y} (4x - y)\, dx dy &= \int_0^2 \left[\frac{4x^2}{2} - yx \right]_{y^2}^{2y} dy \\
&= \int_0^2 8y^2 - 2y^2 - 2y^4 + y^3 \, dy \\
&= \left[\frac{2y^5}{5} + \frac{y^4}{4} + \frac{6y^3}{3} \right]_0^2 = 7\frac{1}{5}
\end{aligned}$$

(ii) DIAGRAM

$$\begin{aligned}
\int_{x=0}^{2} \int_{y=0}^{x^2} y \cos x^5 \, dy dx &= \int_0^2 \left[\frac{y^2}{2} \right]_0^{x^2} \cos x^5 \, dx \\
&= \int_0^2 \frac{x^4}{2} \cos x^5 \, dx
\end{aligned}$$

Putting $u = x^5$, so $\frac{du}{dx} = 5x^4$ we get

$$\int_{x=0}^{2} \frac{\cos u}{10} \, du = \int_{u=0}^{3} 2 \frac{\cos u}{10} \, du = \frac{1}{10} \sin 32$$

(iii) DIAGRAM

$$\int_{y=0}^{3} \int_{x=y^2}^{\frac{y^2}{3}+6} f(x, y)\, dx dy$$