Question
Determine all the circles in $\overline{\mathbf{C}}$ that are taken to themselves by the Möbius transformation $m(z) = \frac{3z-2}{2z-1}$. (That is, determine all the circles A in $\overline{\mathbf{C}}$ satisfying $m(A) = A$.)

Answer
First, find the fixed points and the type of m:
$m(z) = \dfrac{3z-2}{2z-1}$ $\det(m) = -3+4 = 1$, so m is already normalized.
$\underline{\tau(m) = (3-1)^2 = 4 \text{ and so } m \text{ is parabolic.}}$

Fixed point:
$m(z) = z$
$2z^2 - z = 3z - 2$
$2z^2 - 4z + 2 = 0$
$z^2 - 2z + 1 = 0$
$(z-1)^2 = 0$ so m(1) = 1

Since the coefficients of m are real, $m(\mathbf{R}) = \mathbf{R}$. If A is a circle in $\bar{\mathbf{C}}$ intersecting $\mathbf{R}$ in two points (1=fixed point of m and z_0), then $m(A) \neq A$ since $m(z_0) \neq z_0$. If A is a circle in $\bar{\mathbf{C}}$ which is tangent to $\mathbf{R}$ at 1, then $m(A) = A$. So, the circles taken to themselves by m are $\bar{\mathbf{R}}$ and any circle in $\mathbf{C}$ tangent to $\mathbf{R}$ at 1.