QUESTION

Let C denote any simple closed contour taken in the counterclockwise sense and write

$$g(w) = \int_C \frac{z^3 + 2z}{(z - w)^3} dz$$

Show that $g(w) = 6\pi i w$ when w is inside C and $g(w) = 0$ when w is outside C.

ANSWER

In (*) we want $n = 2$, $f(z) = z^3 + 2z$ and $w = b$. $f''(z) = 6z$, so $g(w) = \frac{2\pi i}{2!} 6w = 6\pi i w$ if w lies inside C. If w lies inside C then $g(w) = 0$ by Cauchy's Theorem.