Question
Find and sketch the characteristic curves for

$$2u_{xx} - 5u_{xx} + 2u_{yy} + u_x - 3u = 0$$

For what values of α does it have a unique solution satisfying the conditions

$$u(\cos\theta, \sin\theta) = \theta,\ \ u_x(\cos\theta, \sin\theta) = 0 \text{ for } 0 < \theta < \alpha?$$

Answer
1st order derivatives are irrelevant for classification.
Thus:
$a = 2,\ b = -\dfrac{5}{2},\ \ c = 2$
$b^2 - ac = \left(\dfrac{5}{2}\right)^2 - 4 = \dfrac{9}{4} > 0$ Therefore hyperbolic everywhere
Characteristic equations given by:

$$\frac{Dy}{dx} = \frac{0\frac{5}{2} \pm \sqrt{\frac{9}{4}}}{2} = \frac{1}{2}\left(-\frac{5}{2} \pm \frac{3}{2}\right) = -\frac{1}{2} \text{ or } -2$$

so characteristics or $\begin{cases} y &= -\dfrac{1}{2} + const \\ y &= -2x + const \end{cases}$
i.e., 2 sets of straight lines
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Now we're given boundary conditions as:
$u(\cos\theta, \sin\theta) = \theta,\ \ u_x(\cos\theta, \sin\theta) = 0$ for $0 < \theta < 2\pi$
i.e., on an arc of a circle of radius 1.
Now remember the example of lectures where characteristics carried information from the curve on which boundary conditions are given into the domain of a solution.
Draw first the boundary condition curve. Then superimpose a grid given by the characteristics.
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Now consider the point B. This is where $y = -2x + const$ is first tangential to the circle.
Any point on the circle between A and B, P_1, say, has two characteristics passing through it and into the range of influence. However these characteristics given by $y = -2x + const$ can be seen to encounter the circle at another point in the first quadrant, P_2, say. If we think of characteristics as propagating information from the curve in which the boundary condition is defined into the range of influence (cf. lecture example), then we have a possible conflict at this second intersection point.
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In other words the information propagated from the boundary condition at P_1 may be different from the information given by the boundary condition at P_2.
The only way we can be sure things will work out is if we only define the boundary conditions up to $\underline{B}$ (which turns out to be $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$. Thus we have a domain of dependence.
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Thus $\alpha = \angle AOB = tan^{-1}\left(\frac{1}{2}\right)$.