QUESTION
Reconsider the above question from the point of view of diversification to reduce risk. Does this make sense when the assets are

(a) perfectly positively correlated ($\rho_{12} = 1$);

(b) perfectly uncorrelated ($\rho_{12} = 0$);

(c) the shares are perfectly negatively correlated ($\rho_{12} = -1$).

In the case of (c), show that it is theoreticly possible to obtain a risk-free portfolio.
ANSWER
Reduction of risk is equivalent to minimising σ^2

$$\sigma^2 = \theta_1^2\sigma_1^2 + \theta_2^2\sigma_2^2 + 2\theta_1\theta_2\rho_{12}$$

(a) $\rho_{12} = 1 \Rightarrow$

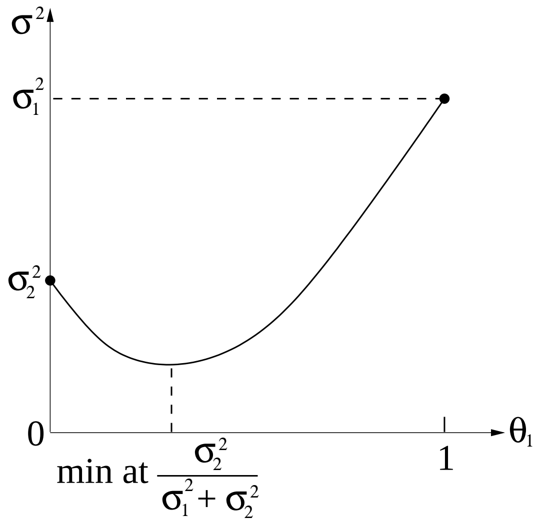
$$\begin{aligned} \sigma^2 &= \theta_1^2\sigma_1^2 + \theta_2^2\sigma_2^2 + 2\theta_1\theta_2 \\ &> \theta_1^2\sigma_1^2 + \theta_2^2\sigma_2^2 \\ &> \sigma_1^2 \text{ or } \sigma_2^2 \end{aligned}$$

Thus it doesn't make sense to diversify into positively correlated assets as the variance (uncertainty) increases. (Or if one goes down, so does the other).

(b) $\rho_{12} = 0 \Rightarrow$

$$\begin{aligned} \sigma^2 &= \theta_1^2\sigma_1^2 + \theta_2^2\sigma_2^2 + 0 \\ &= \theta_1^2\sigma_1^2 + (1-\theta_1)^2\sigma_2^2 \\ &= \theta_1^2(\sigma_1 + \sigma_2) - 2\sigma_2^2\theta_1 + \sigma_2^2 \end{aligned}$$

Which as a plot against θ like:



There is a minimum at $\theta_1 = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2} \Rightarrow \sigma^2$ valueless$<\sigma_2^2$ or σ_1^2 so it is better to diversify $(= \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2})$ optional portfolio.

(c) $\rho_{12} = -1 \Rightarrow$

$$\begin{aligned}
\sigma^2 &= \theta_1^2\sigma_1^2 + \theta_6^2\sigma_2^2 - 2\theta_1\theta_2 \\
&= \theta_1^2\sigma_1^2 + (1-\theta_1)^2\sigma_2^2 - 2\theta_1(1-\theta_1) \\
&= \theta_1^2(\sigma_1^2 + \sigma_2^2 + 2) - \theta_1(2\sigma_2^2 + 2) + \sigma_2^2
\end{aligned}$$