Ordinary Differential Equations
Classification

Question
Find 2 values of r for which $y = e^{rx}$ is a solution of $y'' - y' - 2y = 0$.
Find a solution to satisfy $y(0) = 1$ and $y'(0) = 2$.
Answer
$y = e^{rx}$ is a solution to $y'' - y' - 2y = 0$ if $r^2e^{rx} - re^{rx} - 2e^{rx} = 0$, i.e. $r^2 - 2 - 2 = 0$.
The roots of this quadratic equation are $r = 2$ and $r = -1$.
The DE os linear and homogeneous, so any function of the form

$$y = Ae^{2x} + Be^{-x}$$

is a solution.
The solution will satisfy

$$\begin{aligned} 1 &= y(0) = A + B \\ 2 &= y'(0) = 2A - B \end{aligned}$$

if A and B take the values

$$\begin{aligned} A &= 1 \\ B &= 0 \end{aligned}$$

So the solution is

$$y = e^{2x}$$