Ordinary Differential Equations
Classification

Question
If one solution of $y'' - k^2 y = 0$ is $y_1 = e^{kx}$, guess and verify a solution y_2 that is not a multiple of y_1.
Find a solution to satisfy $y(1) = 0$ and $y'(1) = 2$.
Answer
As $y_1 = e^{kx}$ is a solution. A sensible guess for y_2 is $y_2 = e^{-kx}$.
Since
$$y_2'' - k^2 y_2 = k^2 e^{-kx} - k^2 e^{-kx} = 0$$
then y_2 is confirmed as a solution.
The DE is linear and homogeneous, so any function of the form
$$y = Ay_1 + By_2 = Ae^{kx} + Be^{-kx}$$
is also a solution.
To satisfy the given conditions:
$$\begin{aligned} 0 &= y(1) = Ae^k + Be^{-k} \\ 2 &= y'(1) = Ake^k - Bke^{-k} \end{aligned}$$
provided that
$$\begin{aligned} A &= e^{-k}/k \\ B &= -e^k/k \end{aligned}$$
So the solution is
$$y = \frac{1}{k} e^{k(x-1)} - \frac{1}{k} e^{-k(x-1)}$$