QUESTION Write down th mgf of a Poisson distribution with mean μ. Hence write down the mgf for the standardized Poisson variable. Show that as $\mu \to \infty$ this latter mgf tends to $E^{\frac{1}{2}t^2}$, and hence confirm that a Poisson variable can be approximated by a normal distribution when μ is large.

ANSWER

$$\begin{aligned} M(t) &= \sum_o^\infty e^{xt} e^{-\mu} \frac{\mu^x}{x!} \quad \text{for} \phi(\mu) \\ &= e^{-\mu} \sum_0^\infty \frac{(\mu e^t)^x}{x!} \\ &= e^{-\mu} e \mu e^t \end{aligned}$$

For $\phi(\mu)$ mean $= \sigma^2 = \mu$. Standardized variable $Y = \frac{x-\mu}{\sqrt{\mu}}$

$$\begin{aligned} M_Y(t) &= e^{-\frac{\mu}{\sigma}t} M_x(\frac{t}{6}) \\ &= e^{-\sqrt{\mu}t} e^{-\mu} e^{\mu e^{\frac{t}{\sqrt{\mu}}}} \\ &= e^{-\mu-\sqrt{\mu}t} e^{\mu(1+\frac{t}{\sqrt{\mu}}+\frac{t^2}{2\mu}+\frac{t^3}{6\mu\sqrt{\mu}}+\ldots)} \\ &= e^{\frac{t^2}{2}+\frac{t^3}{6\sqrt{\mu}}} \to e^{\frac{1}{2}t^2} \text{ as } \mu \to \infty \end{aligned}$$

which is the mgf of N(0,1) hence by uniqueness.