QUESTION Find the mgf for a random variable which is uniform on the interval $a < x < b$. Use the mgf to find the mean and variance of the distribution.

ANSWER $f(x) = \frac{1}{b-a}$ $a < x < b$

$$\begin{aligned}
M(t) &= \int_a^b \frac{e^{xt}}{b-a}\,dx \\
&= \frac{1}{t}[\frac{e^{xt}}{b-a}]_a^b \\
&= \frac{e^{bt}-e^{at}}{t(b-a)} \\
&= \frac{1}{t(b-a)}[(1+bt+\frac{(bt^2)}{2!}+\frac{(bt)^3}{3!}+\ldots)-(1+at+\frac{(at)^2}{2!}+(at)^3 3! \\
&= \frac{1}{t(b-a)}[(b-a)t+\frac{t^2}{2!}(b^2-a^2)+\frac{t^3}{3!}(b^3-a^3)+\ldots] \\
&= 1+\frac{t(b+a)}{2}+\frac{t^2}{3!}(b^2+ab+a^2)+\ldots
\end{aligned}$$

μ= the coefficient of t $=\frac{b+a}{2}$ $E(X^2) = 2! \times$ the coefficient of t62$=\frac{b^2+ab+a^2}{3}$ $\sigma^2 = \frac{b^2+ab+a^2}{3} - \frac{(b+a)^2}{4} = \frac{(b-a)^2)}{12}$ Note that differentiation doesn't work well here because of difficulties at t=0.