QUESTION The mgf of a random variable Y is e^{3t+8t^2}. Prove that E(Y)=3 and find Var(Y).

ANSWER $M(t) = e^{3t+8t^2} = 1 + (3t + 8t^2) + \frac{(3t+8t^2)^2}{2!} + \dots$ therefore $\mu =$ the coefficient of t $= 3$
$\frac{E(X^2)}{2!} =$ the coefficient of t$^2 = 8 + \frac{9}{2}$ therefore $E(X^2) = 25$.
Alternatively $M^{(1)} = (3 + 16t)e^{3t+8t^2}$, $\mu = M^{(1)}(0) = 3$
$M^{(2)}(t) = e^{3t+8t^2} + (3 + 16t)^2 e^{3t+8t^2}$, $E(X^2) = M^{(2)}(0) = 16 + 9 = 25$
Alternatively having found $\mu = 3$ $M^*(t) = e^{8t^2}$ we can find σ^2 directly by expansion of differentiation.Note that $M(t) = e^{3t+\frac{1}{2}\times 16t^2} \approx e^{\mu+\frac{1}{2}\sigma^2 t^2}$ for $N(\mu, \sigma^2)$ Hence by uniqueness $\mu = 3$ and $\sigma^2 = 16$