QUESTION A random variable has pgf $\frac{(1+2z)^4}{81}$. Find $p(2)$, $p(4)$, μ and σ^2.

ANSWER

$$
\begin{aligned}
G(z) &= \frac{(1+2z)^4}{81} \\
G^{(1)}(z) &= \frac{8(1+2z)^3}{81} \\
G^{(2)}(z) &= \frac{16(1+2z)^2}{27} \\
2!p(2) &= G^{(2)}(0) = \frac{16}{27} \text{ therefore } p(2) = \frac{8}{27} \\
G^{(3)}(z) &= \frac{64(1+2z)}{27} \\
G^{(4)}(z) &= \frac{128}{27} \\
4!p(4) &= G^{(4)}(0) = \frac{128}{27} \text{ therefore } p(4) = \frac{16}{81}
\end{aligned}
$$

$\mu = G^{(1)}(1) = \frac{8\times 3^3}{81} = \frac{8}{3}$

$E(X(X-1)) = G^{('')}(1) = \frac{16\times 9}{27} = \frac{16}{3}$ therefore $\sigma^2 = \frac{16}{3} + 83 - (\frac{8}{3})^2 = \frac{8}{9}$

Note that $G(z) = (\frac{1}{3} + \frac{2}{3}z)^4$. This corresponds to $B(4,\frac{2}{3})$ Hence $p(2) = \left(\begin{array}{c} 4 \\ 2 \end{array}\right)(\frac{2}{3})^2(\frac{1}{3})^2 = \frac{8}{27}$, $p(4) = (\frac{2}{3})^4 = \frac{16}{81}$.

$\mu = 4 \times \frac{2}{3} = \frac{8}{3}$, $\sigma^2 = 4 \times \frac{2}{3} \times \frac{1}{3} = \frac{8}{9}$