Question

Suppose that X and Y is $N_2(\mu_x, \mu_y, \sigma_x, \sigma_y, \rho)$ for which
$E(X|Y = y) = 3.7 - 0.15y$, $E(Y|X = x) = 0.4 - 0.6x$, $\text{var}(Y|X = x) = 3.64$.
Find all the five parameters.

Answer

We know

$$E(X|Y = y) = \mu_x + \rho\frac{\sigma_x}{\sigma_y}(y - \mu_y)$$

$$E(Y|X = x) = \mu_y + \rho\frac{\sigma_y}{\sigma_x}(x - \mu_x)$$

$$\text{var}(Y|X = x) = \sigma_y^2(1 - \rho^2)$$

Here:

$$E(X|Y = y) = 3.7 - 0.15y$$

$$E(Y|X = x) = 0.4 - 0.6x$$

$$\text{var}(Y|X = x) = 3.64$$

Note that the coefficient of y in $E(X|Y = y)$ is $\rho\frac{\sigma_x}{\sigma_y}$
Note that the coefficient of x in $E(Y|X = x)$ is $\rho\frac{\sigma_y}{\sigma_x}$
Multiplying the two we get ρ^2.
Therefore $\rho^2 = (-0.15)(-0.6) = 0.09$
Therefore $\rho = -\sqrt{0.09} = -0.3$
Negative sign because coefficient of y in $E(X|Y = y)$ is $= -0.15 = \rho\frac{\sigma_y}{\sigma_x}$ and
σ_x and σ_y are positive.

$$\begin{aligned}
\text{var}(Y|X = x) &= \sigma_y^2(1 - \rho^2) = 3.64 \\
&\Rightarrow \sigma_y^2(1 - 0.09) = 3.64 \\
&\Rightarrow \sigma_y^2 = 4
\end{aligned}$$

Now

$$\frac{\rho\sigma_x}{\sigma_y} = -0.15 \quad\Rightarrow\quad \frac{(-0.3)\sigma_x}{2} = -0.15$$

$$\Rightarrow\quad \sigma_x = 1$$

Now $\left.\begin{aligned} \mu_x - 0.15(-\mu_y) &= 3.7 \\ \text{and} \quad \mu_y - 0.6(-\mu_x) &= 0.4 \end{aligned}\right\}$ solve for μ_x, μ_y.

Final answer: $\mu_x = 4$, $\mu_y = 2$, $\sigma_x = 1$, $\sigma_y = 2$, $\rho = -0.3$