Question
Show that the rationals $\mathbf{Q}$ with their usual order $<$ form an ordered field but not a complete ordered field.

Answer
To see that $\mathbf{Q}$ is not a complete ordered field, note that the subset $A = \{a \in \mathbf{Q} \mid a < \sqrt{2}\}$ is bounded above, for instance by $s = 2$, but has no supremum in $\mathbf{Q}$: that is, for every rational number s so that $a \leq s$ for every $a \in A$, we have that there exists another rational number t so that $t < s$ and $a \leq t$ for every $a \in A$. (One way to see this is to use decimal expansions, and to recall that a number is rational if and only if its decimal expansion is either repeating or terminating.)