QUESTION

Show that

(a) $\nabla \times (\nabla \phi) = \mathbf{0}$ (b) $\nabla \cdot (\nabla \times \mathbf{a}) = 0$

ANSWER

(a) $\mathbf{a} = (x^2 y, -2xz, 2yz)$

$\nabla \times \mathbf{a} = (2z - 2x, 0, -2z - x^2)$

$\nabla \times (\nabla \times \mathbf{a}) = (0, 2 + 2x, 0)$

(b) Same vector $\mathbf{a}$, $\nabla \cdot \mathbf{a} = 2xy + 2y$

$\nabla(\nabla \cdot \mathbf{a}) = (2y, 2x + 2, 0)$

$\nabla^2 \mathbf{a} = (2y, 0, 0)$

$\nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a} = (0, 2x + 2, 0)$

This should agree with 2(a) by a general formula!