Question

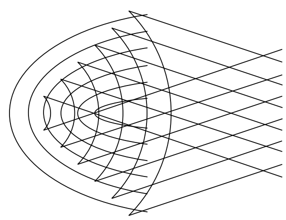
Let $\gamma(t)$ be a regular plane curve, and let d be a positive constant. The curve $\delta(t)$ given by

$$\delta(t) = \gamma(t) + dN(t)$$

is called the *parallel* to γ at distance d. Show that $\delta(t)$ is a regular curve *except* where δ intersects the evolute of γ.
Taking γ to be a parabola, sketch δ for increasing values of d.

Answer

$\delta(t) = \gamma(t) + dN(t)$**, where γ is unit speed, so $\dot{\gamma} = \gamma'$, $\dot{N} = N$.**
$\dot{\delta}(t) = \gamma'(t) + dN'(t) = T - d.KT = (1 - dK)T$**.**
Thus $\dot{\delta} \neq 0$ except when $1 - dK = 0$, i.e. $d = \rho = K^{-1}$;
then $\delta(t) =$ centre of curve $\Rightarrow$ lies on evolute.



Note how the singular points of the parallels trace out the evolute.