QUESTION

(a) An investor wishes to trade in options on an asset whose current price one year from the maturity date of an option is \$25, the exercise price of the option is \$20, the risk-free interest rate is 5% per annum and the asset volatility is 20% per annum. Calculate by what amount the asset price has to change for the purchaser of a European call option to break even giving your answer to 4 decimal places?

(b) Write down the call-put parity formula for European options. Hence repeat part (a) but for a European put.

(c) Sketch the qualitative behaviour of the European call and put values over the lifetime of the option as a function of the underlying asset price.

(d) Calculate the initial price of the call option in part (a) if the asset pays a continuous dividend of DS where S is the asset price and $D = 0.01$.

You may assume that the solution of the Black-Scholes equation for a European call option, paying no dividends, is given by,

$$c(S,t) = SN(d_1) - K\exp(-r(T-t))N(d_2),$$

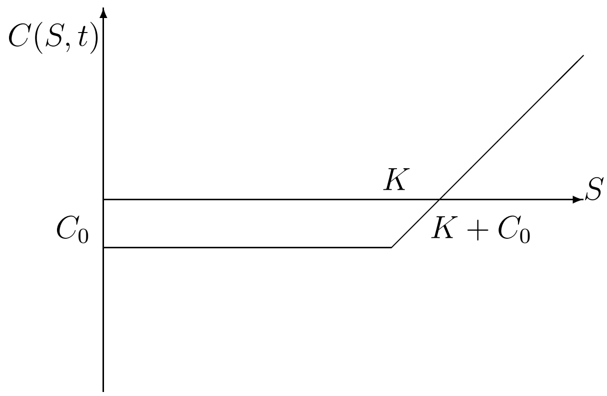
$$d_1 = \frac{\log\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\log\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}.$$

ANSWER

(a) $T = 1,\ S_0 = 25,\ K = 20,\ r = 0.05,\ \sigma = 0.2$

For the holder of a Eurocall, the asset price must rise by the following to break even:



Payoff at $t = T$.

Therefore the price must rise to $K + C$ for the holder to break even. If the initial asset price is S_0, requires final asset price is $K + C_0$ so the rise must be $K + C_0 - S_0$ Therefore we need to know the initial premium at S_0.

Use the formula given at $t = 0$.

$$
\begin{aligned}
C(S_0, 0) &= S_0 N(d_1(0)) - K e^{-rT} N(d_2(0)) \\
d_1(0) &= \frac{\left(\log\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2} T\right)\right)}{\sigma\sqrt{T}} \\
d_2(0) &= \frac{\left(\log\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2} T\right)\right)}{\sigma\sqrt{T}}
\end{aligned}
$$

Feed in the above data to get

$$\left.\begin{array}{l} d_1 = 1.47 \\ d_2 = 1.27 \end{array}\right\} \text{to 2 d.p.}$$

We need to find $N(1.47)$ and $N(1.27)$. From the tables, $N(1.47) = 0.9297$, $N(1.27) = 0.8980$

$$C(S, 0) = 25 \times 0.9292 - 20e^{-0.05} \times 0.8980 = 6.1459$$

Therefore to break even they need a new price of $K + C - 0 = 20 + 6.1459 = 26.1459$

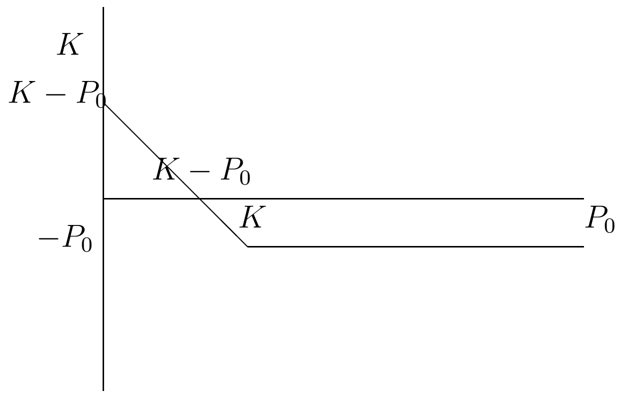
Therefore the current price needs to rise by $K + C_0 - S = 1.1459$.

(b) The call-put parity formula is

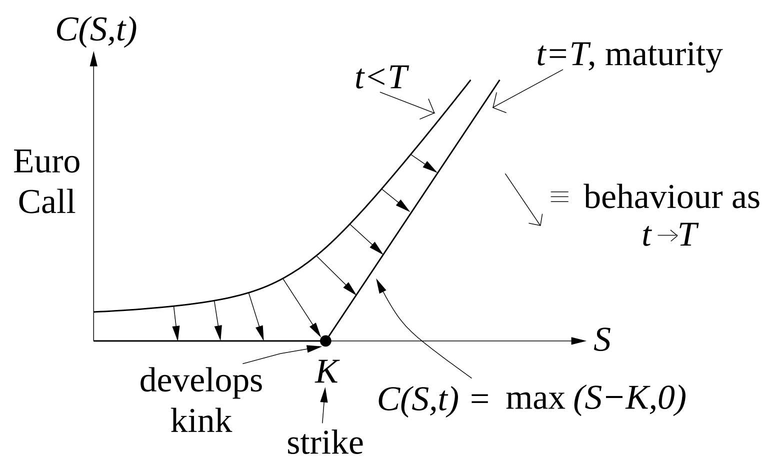
$$C(S, t) - P(S < t) = S - K e^{-r(T-t)}$$

Therefore

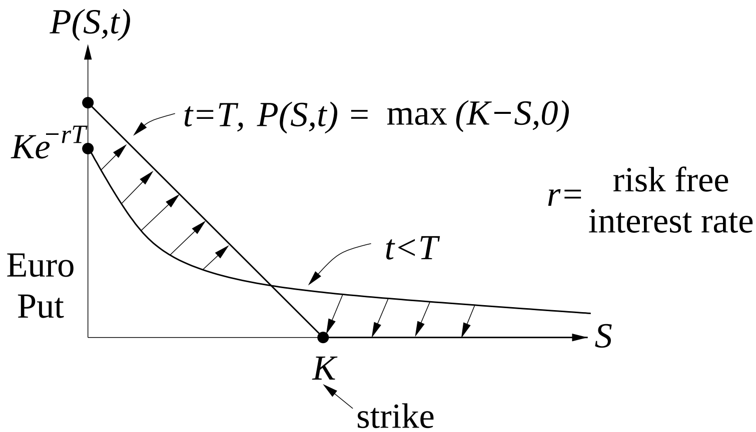
$$
\begin{aligned}
P(S_0, 0) &= C(s_0, 0) - S_0 + Ke^{-rT} \\
&= 1.1459 - 25 + 20e^{-0.05} \\
&= 0.1705
\end{aligned}
$$



So need price to fall to $K - P_0$ to break even $=20 - 0.1705 = 19.8295$ i.e. needs to fall by $25 - 19.8295 = 5.1705$.



(c)



(d) Result of part (a) changes by converting $r \to r - D$ (as per example in lecture notes) in all equations. Thus the effective interest rate is $0.05 - 0.01 = 0.04$

$$d_1(0) = \frac{\left(\log\left(\frac{S_0}{K}\right) + \left(r - D + \frac{\sigma^2}{2}\right)T\right)}{\sigma\sqrt{T}}$$

etc.

With

$$\begin{aligned}
C(S,t) &= Se^{-D(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2) \\
d_1(0) &= 1.42 \\
d_2(0) &= 1.22 \\
C(s_0,0) &= 25e^{-0.01}N(1.42) - 2oe^{-0.05}N(1.22) \\
N(1.42) &= 0.9222 \\
N(1.22) &= 0.8888 \\
C(S_0,0) &= 22.8256 - 16.9091 = 5.9165
\end{aligned}$$