Question

($*$) A car with a mass of 1000kg accelerates down a road with the engine producing a force of $2000 kg\ m/sec^2$. Determine how quickly the car reach $10 m/s$ from stationary and how far it travels in this time.

Now consider extending the model to account for the fact that the car is subjected to wind resistance which produces a resistance force proportional to the speed of the car (the constant of proportionality is 40 (with all measurements in m, kg and sec). How much longer does it take to get to $10 m/sec$ when this air resistance force is accounted for. What is the maximum speed of the car in this case.

Answer

$1000\frac{d^2x}{dt^2} = 2000 \Rightarrow \frac{d^2x}{dt^2} = 2 \Rightarrow x = t^2 + At + B$

$\Rightarrow x(0) = \frac{dx}{dt}(0) = 0 \Rightarrow x = t^2, \quad \frac{dx}{dt} = 2t$

$\frac{dx}{dt} = 10$ when $2t = 10 \Rightarrow t = 5$ sec, at $t = 5$ sec, $\quad x = (5)^2 = 25$metres.

Now with air resistance $1000\frac{d^2x}{dt^2} = 2000 - 40\frac{dx}{dt}$

The linear equation for v is $1000\frac{dv}{dt} + 4v = 200 \Rightarrow \frac{dv}{dt} + \frac{1}{25}v = 2$

$I(x) = \exp\left(\int \frac{1}{25}dt\right) = \exp\left(\frac{t}{25}\right)$

$\Rightarrow \frac{d}{dt}\left(v \exp\left(\frac{t}{25}\right)\right) = 2\exp\left(\frac{t}{25}\right)$

$\Rightarrow ve^{\frac{t}{25}} = 50e^{\frac{t}{25}} + A$

$v(0) = 0 \Rightarrow A = -50$

$v(t) = 50\left(1 - e^{-\frac{t}{25}}\right)$

maximum speed is when $t \to \infty$ and $v \to 50$m/s

$v = 10$ when $10 = 50\left(1 - e^{-\frac{t}{25}}\right) \Rightarrow \ln\left(1 - \frac{1}{5}\right) = -\frac{1}{25}t$

$t = 25\ln\frac{5}{4} \approx 5.578 \Rightarrow$ air resistance slows the car so it takes 0.578 seconds longer to get to 10m/s.