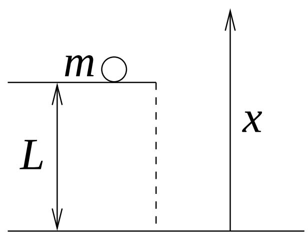
Question

Near the earth the gravitational force is $-mg$ and hence if x measures distance from the earths surface then the gravitational potential is mgx. Hence consider a ball of mass m which rolls off the edge of a table and drops from a height L onto the ground. Using Newton's second law, with only gravity acting, determine the ball's speed when it hits the ground. Derive an expression for the energy of the particle and hence show this predicts the same speed at impact. What is the work done by the ball from the top of the table to the floor.

Answer



$m\frac{d^2x}{dt^2} = -mg$ with $x = L$ at $t = 0$ and $\frac{dx}{dt} = 0$ at $t = 0$

$\Rightarrow \frac{d^2x}{dt^2} = -g \Rightarrow x = -\frac{g}{2}t^2 + At + B$ hence $x = -\frac{g}{2}t^2 + L$

Therefore the ball hits the ground when $x = 0 \Rightarrow 0 = -\frac{g}{2}t^2 + L$

$\Rightarrow t = \sqrt{\frac{2L}{g}}$

at $t = \sqrt{\frac{2L}{g}}$, $\frac{dx}{dt} = -gt = -\sqrt{2gL}$.

So the speed when ball hits floor is $\sqrt{2gL}$.

Energy = kinetic + potential

Kinetic $= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$, Potential $= mgx$

Hence Energy $= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + mgx$ because the force only depends on x (conservative) $\Rightarrow$ Energy is constant.

at $t = 0$, $\frac{dx}{dt} = 0$ and $x = L \Rightarrow$ Energy $= mgL$

Hence for all time $\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + mgx = mgL$

Hence when $x = 0$, $\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = mgL \Rightarrow \left(\frac{dx}{dt}\right)^2 = 2gL$

speed at impact $= \sqrt{2gL}$

Work done $= \int_{x=L}^{x=0} (-mg)dx = [-mgx]_{x=L}^{x=0} = mgL$