QUESTION

Let C_0 denote the circle $|z-z_0|=R$, taken counterclockwise. Prove that

$$\int_{C_0} \frac{dz}{z-z_0} = 2\pi i$$

and

$$\int_{C_0} (z-z_0)^{n-1} ds = 0, \quad (n = \pm 1, \pm 2, \ldots).$$

ANSWER

By question 3 ,

$$\int_{C_0} \frac{dz}{z-z_0} = \int_C \frac{dz}{z} = \int_0^{2\pi} \frac{iRe^{it}dt}{Re^{it}} = 2\pi i.$$

Also.

$$\int_{C_0} (z-z_0)^{n-1} dz = \int_C z^{n-1} dz = \int_0^{2\pi} R^{n-1} e^{n-1} Rie^{it} dt = R^{ni} \int_0^{2\pi} e^{nit} dt$$

(using the substitution $z = Re^{it}$). We thus get $\frac{R^n}{n} i [e^{nit}]_0^{2\pi} = 0$ if $n \neq 0$.