QUESTION

Let C and C_0 denote the circles $z = Re^{it}$, $(0 \le t \le 2\pi)$ and $z = z_0 + Re^{it}$, $(0 \le t \le 2\pi)$. Draw pictures of these circles and use these parametric representations to show that

$$\int_C f(z)dz = \int_{C_0} f(z - z_0)dz.$$

ANSWER

DIAGRAM

By letting $z = z_0 + Re^{it}$ we get $\int_{C_0} f(z - z_0)dz = \int_0^{2\pi} if(Re^{it})Re^{it}dt$. By putting $z = Re^{it}$ we get $\int_C f(z)dz = \int_0^{2\pi} if(Re^{it})Re^{it}dt$. Hence $\int_C f(z)dz = \int_{C_0} f(z - z_0)dz$.